

$$x \neq y$$

1 2

2.2.3

$$S = (a_1 x + b) \pmod{p}$$

$$S = (a_2 x + b) \pmod{p}$$

$$(a_1 x - a_2 x) \pmod{p} = 0$$

$$\frac{(a_1 - a_2) \cdot x}{< p} \pmod{p} = 0$$

$$(a_1 - a_2) \cdot x = i \cdot p$$

$$\frac{x = 0}{a_1 = a_2} \quad \times$$

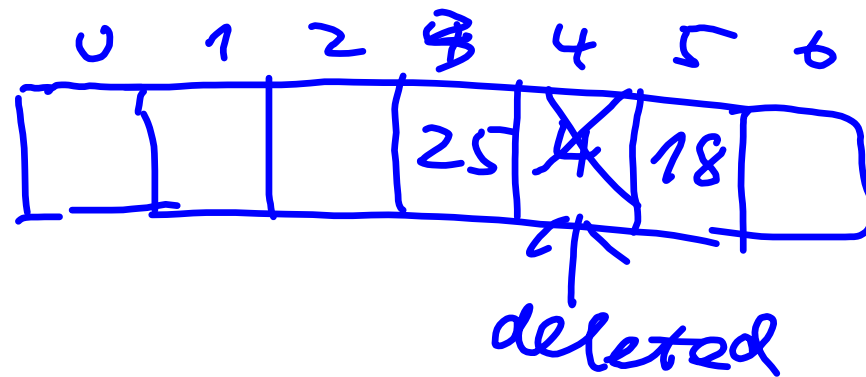
for any valid pair  $(s, t)$   
there is exactly one pair  
 $(a, b), s, t$ .

$$s = (ax + b) \bmod P$$

$$t = (ay + b) \bmod P$$

the number of  
pairs  $(a, b), s, t$   $h_{a,b}(x) = h_{a,b}(y)$

is less than  $\frac{P(P-1)}{m}$



$$\begin{aligned}
 & (h(x) - S(j_1, x)) \bmod m \\
 & (h(x) - S(j_2, x)) \bmod m \\
 & S(j_1, x) - S(j_2, x) = 0 \pmod{m} \\
 & (-1)^{j_1} * \Gamma_2
 \end{aligned}$$

$$(-1)^{j_1} * \left[\frac{j_1}{2}\right]^2 - (-1)^{j_2} * \left[\frac{j_2}{2}\right]^2 \\ = 0 \quad (\text{mod } m)$$

①  $j_1, j_2$  are even.

$$\left[\frac{j_1}{2}\right]^2 - \left[\frac{j_2}{2}\right]^2 = 0 \quad (\text{mod } m)$$

$$\left(\frac{j_1}{2} + \frac{j_2}{2}\right) \cdot \left(\frac{j_1}{2} - \frac{j_2}{2}\right) = 0 \quad (\text{mod } m)$$